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# Strong Coupling Effects in Non-commutative Spaces From OM Theory and Supergravity

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**Abstract**

We show that a four-parameter class of 3+1 dimensional NCOS theories can be obtained by dimensional reduction on a general 2-torus from OM theory. Compactifying two spatial directions of NCOS theory on a 2-torus, we study the transformation properties under the $SO(2,2;Z)$ T-duality group. We then discuss non-perturbative configurations of non-commutative super Yang-Mills theory. In particular, we calculate the tension for magnetic monopoles and (p,q) dyons and exhibit their six-dimensional origin, and construct a supergravity solution representing an instanton in the gauge theory. We also compute the potential for a monopole-antimonopole in the supergravity approximation.

# 1 Introduction

The observation that non-commutative gauge theories can be obtained as limits of string theory in a background $B$ field [1, 2] has led to a number of interesting results, in particular, the emergence of string theories without gravity near certain corners of the moduli space. A simple example is the open string theory constructed from a D3 brane in the presence of a near critical electric field [3, 4]. It is essentially a free string theory with an additional term in the propagator which gives rise to extra phase factors in front of the scattering amplitudes, producing the structure of a non-commutative open string theory (NCOS).

Recently, it was proposed that one can similarly construct a six dimensional theory as the low energy theory of an M5 brane in the presence of a near critical three-form field [5, 6], which contains open membranes and is decoupled from gravity. In view of the parallel picture in M-theory, it appears then natural to conjecture that all these string theories can be obtained as different limits of this six-dimensional (OM) theory [5]. Further discussions can be found in refs. [7] -[21].

The simplest NCOS theory is specified by two parameters, namely the open string coupling $G_s$ and $\alpha'$, where $\alpha'$ represents both the string and the non-commutative scale in the $x_0$-$x_1$ plane. In [13] the NCOS theory was generalized to a larger class of NCOS theories, which are classified by four parameters: $\alpha'$, $G_s$, $\theta^{23}$ and $\chi$ (see [5, 12, 20] for related discussions). The parameter $\theta^{23}$ is the non-commutative scale in the $x_2$-$x_3$ plane, and $\chi$ –which originates from a nonzero value for the RR scalar field– gives rise to a $\theta_{\rm QCD}$ term $\theta_{\rm QCD} \int F\tilde{F}$, $\theta_{\rm QCD} = \chi$, in the low energy Yang-Mills theory. For $\chi$ irrational, it was found that the $SL(2, Z)$ transformations of type IIB superstring theory map the NCOS theory into another one with different parameters. However, when $\chi$ is rational, it was found that there exists an $SL(2, Z)$ transformation which maps the NCOS theory to non-commutative SYM field theory. This generalizes the proposal of [4] that NCOS theory with $\theta^{23} = \chi = 0$ is S-dual to NCSYM theory.

Compactifications of NCOS theories in their own turn lead to various T-duality groups. For the particular case of (3+1) dimensional NCOS compactified on a two torus, we show that the full $SO(2, 2; Z) \simeq SL(2, Z) \times SL(2, Z)$ T-duality group connects NCOS theories. This T-duality group combined with the S-duality $SL(2, Z)$ group yield the usual U-duality group of closed strings on $T^2$; $SL(3, Z) \times SL(2, Z)$. We will argue that this U-duality group can be realized from the OM theory picture by a $T^3$ compactification.

Non-commutative gauge theories contain many interesting non-perturbative dynamical objects (see e.g. [27, 28, 29, 30]). These field-theory configurations can be described in the brane picture e.g. in terms of D-strings or D-instantons (or other branes) intersecting with the D3 brane. In particular, a D-string ending on a D3-brane is a magnetic charge from the point of view of the gauge theory on the brane, located at the end of the string. In this paper we will analyise some of these systems using both weak-coupling string theory and supergravity, thus complementing the discussion of [28, 29].

This paper is organized as follows. In sect. 2 we show that compactification of OM theory on a general 2-torus leads to the 4-parameter class of NCOS theories mentioned above. In section 3 we consider the toroidal compactification of NCOS theory and discuss $SL(2,Z)\times SL(2,Z)$ T-duality transformations. In sect. 4, we discuss supersymmetric solitons and their six-dimensional origin. In sect. 5 we compute the monopole-antimonopole potential in large $N$ non-commutative super Yang-Mills theory.

## 2 Reduction of OM theory on a general torus

In this section we shall consider a reduction on a general torus along an arbitrary oblique direction. This will lead to the general 4-parameter class of NCOS theories in 3+1 dimensions considered in [13]. In [13], the reduction has been done for a rectangular torus, giving 3+1 dimensional NCOS theories with $\alpha', G_s, \theta^{23}$, but $\theta_{\rm QCD} = \chi$ parameter equal to zero. Here we will complete the discussion by considering dimensional reduction on an arbitrary torus, which will permit to incorporate a non-zero parameter $\chi$.

Thus we start with the M5-brane configuration of [5], but we take a general (non-orthogonal) coordinate system in the 2-plane $x_4$-$x_5$, i.e.

$$H_3 = H_{01\tilde{5}}\ dx_0\wedge dx_1\wedge d\tilde{x}_5 + H_{23\tilde{4}}\ dx_2\wedge dx_3\wedge d\tilde{x}_4\ , \tag{2.1}$$

$$\tilde{x}_4 = x_4\cos\gamma + x_5\sin\alpha\ ,\quad \tilde{x}_5 = -x_4\sin\gamma + x_5\cos\alpha\ ,$$

where the coordinates $x_4$ and $x_5$ are periodic with periods

$$x_5 \equiv x_5 + 2\pi R_5\ ,\qquad x_4 \equiv x_4 + 2\pi R_4^0\ . \tag{2.2}$$

The $x_4$ axes is rotated by an angle $\gamma$ with respect to $\tilde{x}_4$, and $x_5$ is the axes which is rotated by $\alpha$ with respect to $\tilde{x}_5$. The axes $\tilde{x}_4,\ \tilde{x}_5$ are orthogonal. The orthogonal rotation considered in [13] is recovered by setting $\gamma = \alpha$.

The metric in tilde coordinates is

$$g_{\mu\nu} = \mathrm{diag}(-\xi^2, \xi^2, \rho^2, \rho^2, \xi^2, \xi^2) \ ,$$

where we have rescaled some coordinates by constant factors $\xi, \rho$, which will be fixed later. So, in the $x_4,\ x_5$ coordinates, the metric for the 45 plane is given by

$$ds^2 = \xi^2(dx_5^2 + dx_4^2 - 2\sin(\gamma - \alpha)dx_4\ dx_5) \ , \tag{2.3}$$

and the $H$ field components are

$$H_{015} = -\xi^3 \tanh\beta \cos\alpha \ , \qquad H_{014} = \xi^3 \tanh\beta \sin\gamma \ , \tag{2.4}$$

$$H_{234} = \rho^2 \xi \sinh\beta \cos\gamma \ , \qquad H_{235} = \rho^2 \xi \sinh\beta \sin\alpha \ . \tag{2.5}$$

The theory should be decoupled from gravity in the limit $\beta \to \infty$, with

$$\begin{gathered} M_{\mathrm{eff}}^3 = \tfrac{1}{2} M_P^3 \Big( \xi^3 + H_{01\tilde{5}} \Big) = \tfrac{1}{2} M_P^3 \xi^3 \Big( 1 - \tanh\beta \Big) = \text{fixed} \ , \\ M_P^3\ \xi\rho^2 = \text{fixed} \ . \end{gathered} \tag{2.6}$$

Thus we set

$$\xi = \xi_0 e^{2\beta/3} \ , \qquad \rho = \rho_0 e^{-\beta/3} \ , \tag{2.7}$$

and scale $\sin\alpha$ so that

$$\sinh\beta\ \sin\alpha = B = \text{fixed} \ . \tag{2.8}$$

with fixed $\xi_0,\ \rho_0,\ B,$ and $M_P$. We see that in the limit $\beta \to \infty$, the angle $\alpha \to 0$.

Next, we make dimensional reduction along $x_5$. This gives $N$ D4 branes with $B_{01},\ B_{23}$ field components, and a non-vanishing RR one-form component $A_\mu$ coming from the 45 component of metric. The gauge field components are

$$\begin{gathered} B_{01} = M_P R_5\ H_{015} = -M_P R_5 \xi^3 \tanh\beta \cos\alpha \ , \\ B_{23} = M_P R_5\ H_{235} = M_P R_5 \rho^2 \xi \sinh\beta \sin\alpha \ , \\ A_4 = (M_P R_5)^{-1}\ \sin(\gamma - \alpha) \ . \end{gathered} \tag{2.9}$$

The other components $H_{014}$, $H_{234}$ lead to non-zero components of the 3-form RR field given by

$$A_{014} = H_{014} \ , \qquad A_{234} = H_{234} \ . \tag{2.10}$$

The 5d metric is

$$g^{(\mathrm{A})}_{\mu\nu} = g_s^{2/3} \mathrm{diag}(-\xi^2, \xi^2, \rho^2, \rho^2, \xi^2 \cos^2(\gamma - \alpha)) \ , \tag{2.11}$$

with

$$g_s = (\xi R_5 M_P)^{3/2} = e^{\beta} (R_5 M_{\mathrm{eff}})^{3/2} \ , \qquad l_s^2 = \alpha' = \frac{1}{M_{\mathrm{eff}}^3 R_5} \ . \tag{2.12}$$

Thus one gets the following electric and magnetic field components:

$$E = B^0{}_1 = \tanh\beta \cos\alpha \ , \qquad B = B^2{}_3 = \sinh\beta \ \sin\alpha \ . \tag{2.13}$$

As $\beta \to \infty$ one gets $B^0{}_1 = 1$ and $B^2{}_3 = B$. Let us write $x_4 = R_4^0 \theta_4$, with $\theta_4 = \theta_4 + 2\pi$. Then

$$g^{(\mathrm{A})}_{44} = (R_4^0)^2 \ M_P R_5 \xi^3 \cos^2(\gamma - \alpha) \ . \tag{2.14}$$

Now we perform T-duality in the $x_4$ direction, which gives $N$ D3 branes with non-vanishing NSNS and RR two-form components, and set

$$\xi_0 = [4 M_P R_5 (1 + B^2)]^{-1/3} \ , \qquad \rho_0 = 2\xi_0 \ .$$

This gives the following closed string metric for the $3+1$ dimensional world-volume:

$$g_{\mu\nu} = \mathrm{diag}(-\frac{1}{1-E^2} \ , \frac{1}{1-E^2} \ , \frac{1}{1+B^2} \ , \frac{1}{1+B^2}) \ ,$$

and an open string metric given by $G_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ . The type IIB closed string coupling constant, $g_s^{(B)}$ and RR scalar field, $\chi$ are

$$g_s^{(B)} = g_s \sqrt{\frac{\alpha'}{g^{(A)}_{44}}} = \frac{M_P R_5 \sqrt{\alpha'}}{R_4^0 \cos(\gamma - \alpha)} \ ,$$

$$\chi = A_4 \ \frac{R_4^0}{\sqrt{\alpha'}} = \frac{R_4^0}{M_P R_5 \sqrt{\alpha'}} \ \sin(\gamma - \alpha) \ . \tag{2.15}$$

Recalling (2.12), i.e. $l_s M_P = 2(1 + B^2)^{1/2}$, we obtain

$$g_s^{(B)} = 2(1 + B^2)^{1/2} \ \frac{R_5}{R_4^0 \cos(\gamma - \alpha)} \ , \qquad \chi = \frac{R_4^0}{2(1+B^2)^{1/2} \ R_5} \ \sin(\gamma - \alpha) \ , \tag{2.16}$$

$$\lambda = \frac{i}{g_s^{(B)}} + \chi = \frac{R_4^0}{2(1+B^2)^{1/2}\ R_5} e^{i(\pi/2+\gamma-\alpha)}\ .$$

Obtaining a non-zero $\chi$ after the limit $\beta \to \infty$ requires a rescaling of the radius $R_4^0$ which is different from that of the $\chi = 0$ case considered in [13]. So it is convenient to discuss these two cases separately.

**I)** $g_s \to \infty$ and $\chi = 0$

This case can be obtained by setting $\gamma = \alpha$, and $\frac{R_4^0}{2(1+B^2)^{1/2}} e^{-\beta} \equiv R_4 = $ fixed. It is worth noting that the compactification radii in the natural OM frame [5] (the coordinate system corresponding to the choice $g_{\mu\nu} = \mathrm{diag}(-\xi^2, \xi^2, \rho^2, \rho^2, \rho^2, \xi^2)$ , for which the natural "open membrane" metric of the low-energy field theory is finite) are both finite, so this is a compactification of OM theory on a finite torus. In this way we reproduce results of [13], namely:

$$G_s = g_s^B \sqrt{1-({B^0}_1)^2}\sqrt{1+({B^2}_3)^2} = 2\frac{R_5}{R_4}(1+B^2) = \text{finite}\ . \tag{2.17}$$

In the $\beta \to \infty$ limit, the type IIB gauge fields are as follows:

$$\begin{aligned} {B^0}_1 = 1\ , \quad & {B^2}_3 = B\ , \quad \chi = 0\ , \\ {A^0}_1 = 0\ , \quad & {A^2}_3 = \frac{1}{G_s}(1+B^2)\ . \end{aligned} \tag{2.18}$$

**II)** $g_s \to \infty$ and $\chi = $ finite

From eq. (2.16) (and recalling that $\alpha \to 0$) we see that in order to have a finite, non-zero value for $\chi$ in the large $\beta$ and large $g_s$ limit, the angle $\gamma$ should approach $\frac{\pi}{2}$ and $R_4^0$ should be kept fixed. This is achieved by scaling these parameters as follows:

$$\frac{\pi}{2} - \gamma = 2\gamma_0\ e^{-\beta}\ , \tag{2.19}$$

$$\frac{R_4^0}{M_P l_s} \equiv R_4 = \text{finite}\ , \tag{2.20}$$

with fixed $\gamma_0$. Then we find

$$g_s^{(B)} = \frac{R_5}{R_4}\frac{e^\beta}{B+\gamma_0}\ , \quad \chi = \frac{R_4}{R_5}\ , \tag{2.21}$$

and hence the open string coupling for the non-commutative theory is

$$G_s = \frac{R_5}{R_4}\frac{1+B^2}{B+\gamma_0}\ . \tag{2.22}$$

The other RR gauge fields (in the $\beta \to \infty$ limit) are

$$A^0{}_1 = A^0{}_{14}\ (M_P R_4) = -\frac{R_4}{R_5} \tanh\beta \sin\gamma = -\chi\ , \tag{2.23}$$

$$A^2{}_3 = A^2{}_{34}\ (M_P R_4) = \frac{R_4}{R_5} \sinh\beta \cos\gamma = \frac{R_4}{R_5}\gamma_0 = \frac{1+B^2}{G_s} - \chi\ B\ .$$

This exactly agrees with the asymptotic values of the gauge fields for the corresponding supergravity configuration for any given $\chi$ (cf. eqs. (3.2)–(3.4) in [13]). The volume of compactification torus in the natural OM theory frame (see above) is proportional to $\frac{\xi}{\rho} \times \cos(\gamma - \alpha)$, which in the $\beta \to \infty$ limit is finite.

The fact that NCOS theories can be obtained by compactification of OM theory on a 2-torus provides a geometric interpretation of the $SL(2, Z)$ symmetry as the modular group of the 2-torus. This is clear before taking the $\beta \to \infty$ limit. Modular transformations of the torus lead to $SL(2, Z)$ transformations of type IIB theory, which in turn produce the transformations of $E$ and $B$ described in [13]. In the limit $\beta \to \infty$, one has $E \to 1$ and $B$ fixed, and one gets the $SL(2, Z)$ transformations of NCOS theories given in [13] (further discussions on $SL(2, Z)$ transformations of NCOS theories can be found in [20]).

# 3 NCOS Theory on a Torus and T-duality

Non-commutative super Yang-Mills theory on a general $T^2$ torus, unlike its commutative counter-part, enjoys the full $SO(2, 2; Z)$ T-duality group of the underlying string theory [2]. In the gauge theory language this is due to the "Morita equivalence", which is an equivalence for the gauge bundles on the non-commutative torus, with a proper mapping between the corresponding gauge groups and couplings, background magnetic fluxes, and volumes of the two tori [1, 31].

Let $x_2, x_3$ be periodic coordinates, $x_2 = x_2 + 2\pi R_2$, $x_3 = x_3 + 2\pi R_3$. The torus volume is $V = 4\pi^2 R_2 R_3$. When the dimensionless non-commutative parameter $\Theta = \theta^{23}/V$ is a rational number, the theory is equivalent to an ordinary SYM in the presence of a magnetic flux [1]. This equivalence can be understood from the T-duality symmetry of type II string theory.

Because the NCOS theories too arise as a limit of type IIB string theory in some background, there will be T-duality connections when they are compactified on a two-torus. Type

IIB string theory has in addition the strong-weak $SL(2,Z)$ duality symmetry. In ref. [13] we described how the S-duality $SL(2,Z)$ transformations act on the parameters of the (3+1) dimensional NCOS theories. The full U-duality group of type IIB superstring on a 2-torus is $SL(3,Z) \times SL(2,Z)$. General type IIB $SL(3) \times SL(2)$ transformations will induce some transformations between different NCOS theories, i.e. it will map one into another with transformed parameters. In view of the results of section 2, NCOS theory on a two-torus can be obtained as a limit of OM theory compactified on a 3-torus $T^3$. The $SL(3,Z)$ part of the U-duality symmetry can then be interpreted geometrically as the modular group of the 3-torus.

NCOS theories on a rectangular two-torus are specified by parameters:

$$(\alpha',\ G_s,\ \theta^{23},\ \chi\ ;\ N,\ R_1,\ R_2)$$

where $\chi$ arises from the expectation value of the RR scalar, giving rise to a $\theta_{\rm QCD}$ term in the low energy effective lagrangian. If $\chi$ is rational, it was shown that the theory is equivalent (by an $SL(2,Z)$ S-duality transformation) to NCSYM theory [13]. In particular, if $\chi = 0$, the NCOS theory with $\theta^{23} = 0$ is S-dual to NCSYM theory by the simple S-duality transformation $\lambda \to -1/\lambda$ that inverts the gauge coupling [4]. Thus NCOS theory with $\chi = \theta^{23} = 0$ can be viewed as the strong coupling limit of NCSYM theory. For the NCOS theory on the torus, the non-commutativity in the $x^0$-$x^1$ directions is characterized by the parameter $\theta^{01} = 2\pi\alpha' = V\Theta$. Although being a perturbative symmetry, T-duality is believed to hold even for strong coupling. This indicates that NCOS theory with rational $2\pi\alpha'/V$ and $\chi = 0 = \theta^{23}$ must be equivalent to an ordinary Yang-Mills theory with a flux.

The precise background fields can be found by transforming the supergravity dual background. One can start with the supergravity background describing NCSYM theory [22, 23] and apply a suitable T-duality transformation leading to a geometry $AdS_5 \times S^5$ with constant dilaton and constant $B_{23}$ field [24]. By S-duality, this can be converted into $AdS_5 \times S^5$ with constant dilaton and constant $A_{23}$ field. Alternatively, one can start with the supergravity dual background describing NCOS theory [23] and look for an $SL(3,Z)$ transformation that leads to $AdS_5 \times S^5$ with constant dilaton. The S-duality and T-duality $SL(2,Z)$ matrices can be embedded into $SL(3,Z)$ matrices as follows:

$$g_S = \begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix} , \qquad g_T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & a & b \\ 0 & c & d \end{pmatrix} .$$

The transformation that takes NCOS theory with rational $\Theta$ into ordinary YM theory is of the form

$$g_U = g_S \; g_T \; g_S = \begin{pmatrix} a & 0 & b \\ 0 & 1 & 0 \\ c & 0 & d \end{pmatrix} , \tag{3.1}$$

where $g_S$ is as above with $a = d = 0$, $c = -b = 1$. Then, the transformed dilaton is constant provided

$$a + bA_{23}^{\infty} = 0 ,$$

where $A_{23}$ is the non-vanishing component of the RR two form. This implies a rational asymptotic value for the $A_{23}$ field, $A_{23}^{\infty} = -\frac{b}{a}$. We omit the details of the calculation (the transformation properties of type IIB supergravity fields under $SL(3)$ can be found in [25]). Thus the required transformation is an element of $SL(3,Z)$ which is not in $SL(2,Z)_T$ or $SL(2,Z)_S$ subgroups. This is already clear from eq. (3.1).

Let us now consider T-duality transformations on NCOS theories with $\theta^{23} \neq 0$ (an independent discussion has appeared in [26] while this paper was in preparation). It should be noted that, for irrational $\chi$, NCOS theories are not related to a NCSYM theory, so to study T-duality of such NCOS theories is convenient to start with the appropriate type IIB configuration and then take a limit leading to a NCOS theory. Once the moduli parameters of the compactified theory are specified, it is easy to obtain the T-duality transformation properties. For the case of our interest, (3+1) dimensional NCOS with parameters, $(\alpha',\ G_s,\ \theta^{23},\ \chi)$, we consider the compactification of the $\theta$-plane $x_2$-$x_3$ on a non-commutative torus two torus $T^2_\theta$. This theory can be realized as some particular limit of type IIB string theory in the presence of a $(D3,(F,D1))$ -brane system compactified on the two torus. We denote the complex and Kahler parameters of that torus by $\tau$ and $\rho$, respectively. Then the brane bound state is characterized by two integers $m,\ N$, whose ratio is proportional to the RR charge density corresponding to D-strings [32, 33].

We choose coordinates so that the components of the closed string metric parallel to the brane bound state are $(-\frac{1}{1-E^2},\ \frac{1}{1-E^2},\ 1,\ 1)$. Along the lines of [33, 34], the spectrum of the open strings attached to the brane bound state is

$$\alpha' M^2 = \frac{|r+q\tau|^2}{\tau_2}\ \frac{\rho_2}{|m+N\rho|^2} + \text{Oscil.}\ , \tag{3.2}$$

where $\tau_2$ and $\rho_2$ are the imaginary parts of $\tau$ and $\rho$, $r$ and $q$ are two integer parameters representing the winding and momentum modes of open strings, respectively. We see that the zero mode part of the spectrum is manifiestly invariant under the T-duality group $SO(2,2;Z) \sim SL(2,Z)_\tau \times\ SL(2,Z)_\rho$. The other open string parameters, i.e. $\theta^{\mu\nu}$ and $G_s$ are [13]

$$\theta^{01} = -\theta^{10} = 2\pi\alpha' E \qquad \theta^{23} = -\theta^{32} = 2\pi\alpha' \frac{B}{1+B^2}\ , \tag{3.3}$$

$$G_s = g_s\sqrt{(1-E^2)(1+B^2)}\ . \tag{3.4}$$

Let us now take the $E \to 1$ limit while keeping $\alpha'$, $G_s$ and the volume of the torus fixed. This leads to a NCOS theory on $T^2_\theta$ defined by parameters: $(\alpha',\ G_s,\ \theta^{23},\ \chi; m,\ N,\ R_1,\ R_2)$. The $SL(2,Z)_\tau$ part consists of transformations under which $\theta^{23},\ m,\ N,\ \chi$ and $G_s$ are invariant; it only acts on the torus metric (and $r$ and $q$ modes). Other transformations are generated by $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,Z)_\rho$, which act on the torus volume $V,\ \theta,\ G_s$ and $(m,\ N)$ as [2]

$$V' = V\ (a+b\Theta) \quad , G_s' = G_s\ (a+b\Theta)\ , \qquad \Theta = \frac{\theta^{23}}{V}\ ,$$

$$\Theta' = \frac{c+d\Theta}{a+b\Theta}\ , \tag{3.5}$$

$$\begin{pmatrix} m' \\ N' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} m \\ N \end{pmatrix}\ .$$

Thus, under this transformation, a NCOS theory is mapped into another NCOS theory with the same $\alpha'$ and $\chi$ parameters, while all other moduli are transformed as above. For the special case of rational $\Theta$ there is a T-duality under which $\Theta$ vanishes and the resulting theory is NCOS theory with $\theta^{23} = 0$.

# 4 Solitons of NCSYM theories from D-branes

Soliton solutions of NCSYM theories can be described in terms of brane configurations in string theory. In the usual (commutative) case, the (3+1) SYM theory can be realized as the low-energy worldvolume theory of a D3-brane (we assume the D3 brane lies along 0123 directions). From this point of view, the BPS monopole solution is described by a D-string ending on the D3-brane, e.g. a D-string along the 04 direction. This picture can be generalized to obtain a description of monopoles in non-commutative super Yang-Mills theories by considering a D3-brane along 0123 directions with a $B_{23}$ field turned on. The worldvolume theory of this brane is then NCSYM with non-commutative parameter $\theta_{23}$. As noted in [28, 29] the requirement of unbroken supersymmetry determines the incident angle of the D1 string on the D3 brane. A part of the present discussion has already appeared in [28]. In addition, we give an explicit solution for the dyonic case, and discuss the M-theory origin of monopole and dyon configurations.

## 4.1 Monopoles and dyons in non-commutative theories

Let $Q_L$ and $Q_R$ be the 32 conserved supercharges of type IIB theory. The D3-brane with a $B_{23}$ field preserves 16 supercharges, given by the supercurrent

$$\epsilon_L\, Q_L + \epsilon_R\, Q_R\ ,$$

where $\epsilon_L$ and $\epsilon_R$ are 16 component Killing spinors which satisfy [35]

$$\Gamma^{0123}\Big(\frac{1}{\sqrt{1+B^2}} - \frac{B}{\sqrt{1+B^2}}\Gamma^{23}\Big)\epsilon_L = \epsilon_R\ . \tag{4.1}$$

In the above equation $\Gamma$ matrices are the 10 dimensional Dirac matrices*:

$$\{\Gamma^\mu,\ \Gamma^\nu\} = 2\eta^{\mu\nu}\ .$$

Let us now add the D-string to this brane system. Suppose that the D-string is along direction $4'$, which makes an angle $\phi_0$ with respect to the $X^4$ direction in the 14 plane, then the unbroken supersymmetry generators are given by Majorana-Weyl spinors satisfying

$$\Gamma^{04} e^{\phi_0 \Gamma^{14}}\epsilon_L = \Gamma^{04}(\cos\phi_0 - \sin\phi_0\Gamma^{14})\epsilon_L = \epsilon_R\ . \tag{4.2}$$

*We choose conventions in which the closed string metric is $\eta_{\mu\nu}$, with signature $(-,+,+,...)$.

In order that eqs.(4.1) and (4.2) have simultaneous solutions, i.e. the BPS condition, we need that

$$\Gamma^{1234}(\cos\phi_0 + \sin\phi_0\Gamma^{14})(\frac{1}{\sqrt{1+B^2}} - \frac{B}{\sqrt{1+B^2}}\Gamma^{23})\epsilon_L = -\epsilon_L \ . \tag{4.3}$$

Since $\Gamma^{14}$ and $\Gamma^{23}$ are commuting, in a proper representation they can be chosen as

$$\begin{aligned}\Gamma^{12} &= i\,\mathrm{diag}(+\mathbf{1}_8, -\mathbf{1}_8),\\ \Gamma^{34} &= i\,\mathrm{diag}(+\mathbf{1}_4, -\mathbf{1}_4, +\mathbf{1}_4, -\mathbf{1}_4)\ ,\end{aligned} \tag{4.4}$$

where by $\mathbf{1}_D$ we denote $D$ dimensional identity matrices. Then eq. (4.3) has solutions provided:

$$\tan\phi_0 = \pm B\ . \tag{4.5}$$

Therefore the D-string should be tilted with respect to D3-brane [28]. The + and − signs correspond to monopole and anti-monopole respectively. This is indeed the same condition found by field theory arguments [29]. Let us now find the monopole mass, which is related to the string tension. If we denote the tension of open F-strings ending on the NC D3-brane by $\frac{1}{\alpha'}$, then, according to the picture of [29], the "shadow" or projection of a D-string on the D3-brane worldvolume carries an energy per unit length $t_{\text{eff}}$ given by

$$t_{\text{eff}} = \frac{T_{\text{D-string}}}{\sin\phi_0} = \frac{1}{\alpha' g_s}\ \frac{1}{\sin\phi_0}\ . \tag{4.6}$$

The appearance of the sine factor is a consequence of the D-string tilt. Using eq.(4.5) we have

$$t_{\text{eff}} = \frac{B}{\alpha' g_s\sqrt{1+B^2}} = \frac{1+B^2}{\alpha' B}\ \frac{1}{g_s\sqrt{1+B^2}} = \frac{1}{g_{YM}^2\theta}\ , \tag{4.7}$$

which is in exact agreement with the results of [29].

One can similarly describe NCSYM $(p,q)$ dyons (with (0,1) corresponding to a D-string). Along the arguments of [36, 37], the $(p,q)$-string can be realized in string theory as a D-string with a non-zero electric background, $E$, where

$$\frac{E}{\sqrt{1-E^2}} = \frac{p}{q}\ g_s\ . \tag{4.8}$$

Then, the supersymmetry preserved by a $(p,q)$-string along the $4'$ direction, instead of eq. (4.2), is given by

$$\Gamma^{04'}(\frac{1}{\sqrt{1-E^2}} - \frac{E}{\sqrt{1-E^2}}\Gamma^{04'})\epsilon_L = \epsilon_R\ , \tag{4.9}$$

where

$$\Gamma^{4'} = \Gamma^4(\cos\phi - \sin\phi\Gamma^{14}) \ . \tag{4.10}$$

Eqs.(4.1) and (4.9) will have simultaneous solutions provided the matrix $A$:

$$A = \Gamma^{1234}(\frac{1}{\sqrt{1-E^2}} - \frac{E}{\sqrt{1-E^2}}\Gamma^{04'})(\cos\phi + \sin\phi\Gamma^{14})(\frac{1}{\sqrt{1+B^2}} - \frac{B}{\sqrt{1+B^2}}\Gamma^{23}) \ , \tag{4.11}$$

has some eigenvalues equal to $-1$. By straightforward matrix algebra we find that this condition is satisfied only when

$$\sin\phi = \frac{B}{\sqrt{1+B^2}} \ \sqrt{1-E^2} \ , \tag{4.12}$$

where $E$ is related to $(p,q)$ as in eq. (4.8). We see that the incident angle for a $(p,q)$ string is in general *less* than that of a D-string. The angle $\phi$ does not depend on the sign of $p$, and only the combination $(\frac{p}{q})^2$ appears in (4.12). In the special cases $p=0$ we recover the monopole case considered above. The case $B=0$ corresponds to $(p,q)$-dyons of a commutative SYM theory.

The effective tension of a $(p,q)$-dyonic state of NCSYM theory will be given by

$$t_{(p,q)} = \frac{T_{(p,q)}}{\sin\phi} = \frac{q}{g_{YM}^2\theta} \ (1 + (\frac{p}{q})^2 g_s^2) \ , \tag{4.13}$$

where the closed string coupling $g_s$ is related to $g_{YM}$ and $\theta$ by $g_{YM} = g_s\sqrt{1+B^2}$ and $\theta = \frac{2\pi\alpha' B}{1+B^2}$.

The present treatment makes use of the perturbative ($g_s << 1$) description of D-branes in terms of Dirichlet (and Neumann) boundary conditions on open strings. The brane intersections discussed here are BPS, so they exist also in the regime $g_s \gg 1$. In the strong coupling limit, one can try to use the S-dual picture, corresponding to NCOS theory with parameter $\theta^{01}$. For the special case of $4+1$ dimensional NCSYM theory, which in the strong coupling limit is OM theory, the $(p,q)$ dyon state corresponds to a (D2-brane, F-string) bound state intersecting a D4-brane. In OM theory, the $(p,q)$ dyon state corresponds to open membrane states ending on the critical M2-M5 brane bound state.

## 4.2 Description of monopoles and dyons from six dimensions

To describe monopole states of (3+1) dimensional NCSYM theory from OM theory, we start with an open membrane ending on M5-brane in the presence of a $H$ field, make dimensional reduction and T-duality. We will follow the conventions of section 2.

The discussion of the previous section regarding the supersymmetry preserved by the D3-D1 bound state can be easily generalized to this case of an M5-brane with a background $H$-field. This system can be viewed as a non-marginal bound state of (M2,M5)-branes [38, 39], and in this section we use their conventions for $\Gamma$ matrices. A similar discussion is given in [40]. Let $\epsilon$ be the asymptotic value of the 32 component Killing spinor of an eleven-dimensional supergravity solution corresponding to a (M5-M2) bound state, describing a M5-brane along 012345 directions with a non-zero $H_{234}$ field. We have

$$\Gamma^{012345}(\frac{1}{\sqrt{1+H^2}} - \Gamma^{234}\frac{H}{\sqrt{1+H^2}})\epsilon = \epsilon \ ,$$

where $H \equiv H_{234}$. The above equation can also be expressed in terms of $H_0 \equiv H_{015}$,

$$\Gamma^{012345}(\frac{1}{\sqrt{1-H_0^2}} + \Gamma^{015}\frac{H_0}{\sqrt{1-H_0^2}})\epsilon = \epsilon \ ,$$

provided that $H_0 = \frac{H}{\sqrt{1+H^2}}$, which is nothing but the *self-duality* condition. The space of solutions of these equations is 16 dimensional, i.e. the configuration preserves $\frac{1}{2}$ of the original supersymmetries. Now we add an intersecting open membrane to the above (M5-M2) bound state. Assuming this membrane lies on the $046'$ directions, with $6'$ being a direction on the 16 plane making an angle $\phi_0$ with respect to 6, the Killing spinor should satisfy

$$\Gamma^{046}(\cos\phi_0 - \Gamma^{16}\sin\phi_0)\epsilon = \epsilon \ .$$

The condition for having a BPS brane intersection tells us that

$$\tan\phi_0 = \pm H \ ,$$

which in terms of the $\beta$ parameter of OM theory is

$$\tan\phi_0 = \pm\sinh\beta \ . \qquad (4.14)$$

One can also consider a more general case in which the open membrane is along $04'6'$. In this case the supersymmetry conditions lead to (4.16).

To obtain a D-string (monopole) intersecting a D3-brane in the presence of a $B_{23}$ field, we make dimensional reduction and T-duality as in section 2, setting both $\alpha$ and $\gamma$ to zero. Reducing along $x^5$, we will find a type IIA configuration of D4-branes along 01234 with a non-zero $B_{23}$ field. The open membrane becomes a D2-brane along $046'$. By T-duality along $x^4$, we end up with a type IIB D3-brane along 0123, with a non-zero $B_{23}$ and a D-string along $06'$, which is precisely the monopole state of the previous section.

The dyonic $(p,q)$ states of a $(3+1)$ NCSYM theory can also be realized from OM theory. Consider an M2-brane lying along $04'6'$, with $4'$ and $6'$ being arbitrary directions in 45 and 16 planes making angles $\psi$ and $\phi$ with respect to 4 and 6 directions, respectively. Reducing this membrane along the 5 direction leads to a (D2-brane,F-string) bound state of type IIA string theory, lying on the $046'$ direction (with the F-strings oriented along the $06'$ directions). The dyonic $(p,q)$ state of $(3+1)$ NCSYM theory is finally obtained upon T-duality along $x^4$ direction. In this way one finds that $\frac{p}{q}$ is related to $\psi$ as

$$\tan\psi = \frac{p}{q} g_s \ . \tag{4.15}$$

Summarizing, the dyon state of NCSYM theory can be described in eleven dimensions by an M5-brane with a non-zero $H_{234}$ field component (which by self-duality requires as well a $H_{015}$ component) with an open membrane along $04'6'$ directions ending on the M5 brane. For a given $\psi$, the supersymmetry conditions fix the angle $\phi$ to be given by

$$\sin\phi = \tanh\beta\cos\psi \ . \tag{4.16}$$

## 4.3 Supergravity solution

Let us first recall the supergravity solution that is dual to large $N$ NCOS theories [23]. We follow the notation of eq. (5.2) of [13].

$$ds^2 = \alpha' f^{1/2}\Big[\frac{u^4}{R^4}(-dx_0^2 + dx_1^2) + \hat{h}\frac{u^4}{R^4}(dx_2^2 + dx_3^2) + du^2 + u^2 d\Omega_5^2\Big] \ , \tag{4.17}$$

$$B_{01} = -\alpha'\frac{u^4}{R^4} \ , \qquad B_{23} = \alpha'\frac{u^4}{R^4}\hat{h}\tan\alpha \ .$$

In ref. [2] it was shown that open strings in the presence of a constant gauge field $B$ and a *flat* metric $g_{\mu\nu}$ can be effectively described as open strings in a background with $B = 0$, but with a metric $G_{\mu\nu}$ and non-commuting coordinates with $[x^\mu, x^\nu] = i\theta^{\mu\nu}$, related to $g_{\mu\nu}$ and $B$ by

$$(G+\theta)^{\mu\nu} = ((g+B)^{-1})^{\mu\nu} \ .$$

In type IIB superstring theory, the presence of D3 branes and gauge fields are sources for gravitational fields, which curve the metric and modify other background fields, e.g. as in (4.17). It is interesting to extend the above definition of $G_{\mu\nu}$ and $\theta^{\mu\nu}$ –which are natural objects from the point of view of the dual non-commutative theory– to this case. We find

$$G^{\mu\nu} = \alpha'^{-1} f^{1/2}\ \eta^{\mu\nu}\ , \tag{4.18}$$

$$\theta^{01} = \alpha'\ \ ,\qquad \theta^{23} = \alpha' \tan\alpha\ . \tag{4.19}$$

Interestingly, although the $B$ field is $u$-dependent, $\theta^{\mu\nu}$ is constant. Unlike $g_{\mu\nu}$, the open string metric is conformal to the $d=4$ Minkowski metric.

A supergravity background representing a D-string intersecting a D3 brane in the presence of a $B$-field can be constructed as follows. For $B=0$, one has the 1/4 supersymmetric solution representing orthogonal intersection of D-string and D3 brane, given by

$$\begin{gathered} ds^2 = f_1^{-\frac{1}{2}}\Big( f_3^{-\frac{1}{2}}[-dx_0^2 + f_1(dx_1^2+dx_2^2+dx_3^2)] + f_3^{\frac{1}{2}}[dx_4^2 + f_1(dr^2 + r^2 d\Omega_4^2)]\Big)\ , \\ e^{2\phi} = g^2 f_1\ ,\quad f_{1,3} = 1 + \frac{Q_{1,3}}{r^3}\ , \\ F_{04r} = \frac{1}{g}\ \partial_r f_1^{-1}\ ,\qquad F_{0123r} = \frac{1}{g}\ \partial_r f_3^{-1} \end{gathered} \tag{4.20}$$

Now we make a T-duality transformation in the $x_2$ direction, a rotation in the plane $(x_2, x_3)$, and T-duality in the new $x_2$ coordinate. We obtain

$$\begin{gathered} ds^2 = f_1^{-\frac{1}{2}}\Big( f_3^{-\frac{1}{2}}[-dx_0^2 + f_1 dx_1^2 + f_1 h(dx_2^2+dx_3^2)] + f_3^{\frac{1}{2}}[dx_4^2 + f_1(dr^2 + r^2 d\Omega_4^2)]\Big)\ , \\ e^{2\phi} = g^2 f_1 h\ ,\quad h^{-1} = \sin^2\theta\ \frac{f_1}{f_3} + \cos^2\theta\ , \\ B_{23} = \frac{\sin\theta}{\cos\theta}\ \frac{f_1}{f_3} h\ , \\ F_{04r} = \frac{1}{g}\ \cos\theta \partial_r f_1^{-1}\ ,\qquad F_{0123r} = \frac{1}{g}\ \cos\theta\ h\ \partial_r f_3^{-1}\ ,\qquad F_{0234r} = \frac{1}{g}\ \sin\theta\ \ h\ \partial_r f_3^{-1}\ . \end{gathered} \tag{4.21}$$

The charge density of the string is distributed along the axes $x_4$ ; the charge density of the D3 brane is distributed in the volume $(\tilde{x}_1, x_2, x_3)$, where

$$\tilde{x}_1 = x_1\cos\theta + x_4 \sin\theta\ .$$

Thus the D-string is at an angle $\frac{\pi}{2}-\theta$ with respect to the D3 brane. The asymptotic value of the $B$-field is $B_{23}^{\infty}=\tan\theta$. This is consistent with the field theory analysis, and with the discussion of supersymmetry of section 4.1.

Let us now consider the decoupling limit. Let $x_4$ be a periodic coordinate $x_4=L\theta_4$, with $\theta_4=\theta_4+2\pi$, and write

$$f_1=1+\frac{Q_1}{r^3}\ ,\qquad f_3=1+\frac{\alpha'^2R^4}{2\pi Lr^3}$$

Now we scale variables as follows

$$r=\alpha' u\ ,\qquad L=\alpha'\tilde{L}\ ,\quad \tan\theta=\frac{\tilde{b}}{\alpha'}\ ,$$

$$x_{2,3}=\alpha'\tilde{x}_{2,3}\ ,\quad Q_1=\alpha'^3\tilde{Q}_1\ ,\quad Q_3=\frac{\alpha'^2R^4}{2\pi L}=\alpha' R_0^4\ ,$$

and take the limit $\alpha'\to 0$ with fixed $u,\tilde{b},R_0,\tilde{L},\tilde{x}_{2,3}$. The resulting metric has following the form

$$\begin{aligned} ds^2=\alpha' f_1^{\frac{1}{2}}\Big[&\frac{u^{\frac{3}{2}}}{R_0^2}[-f_1^{-1}dx_0^2+dx_1^2+\hat{h}(dx_2^2+dx_3^2)]\ ,\\ &+\frac{R_0^2\tilde{L}^2}{f_1u^{\frac{3}{2}}}d\theta_4^2+\frac{R_0^2}{u^{\frac{3}{2}}}(du^2+u^2d\Omega_4^2)\Big] \end{aligned}$$

with

$$\hat{h}^{-1}=1+a^3u^3\ ,\qquad a^3=\frac{\tilde{b}^2}{\tilde{b}^2\tilde{Q}_1+R_0^4}\ .$$

## 4.4 Instanton solution

Finally, we point out another solution in Euclidean space which may be relevant to the study of instantons in non-commutative super Yang-Mills theories [27]. Consider the Euclidean solution representing a bound state of D(-1) and D3 brane. It is given by [41]

$$\begin{aligned} ds^2&=H^{1/2}\Big[f^{-1/2}[dx_0^2+dx_1^2+dx_2^2+dx_3^2]+f^{1/2}(dr^2+r^2d\Omega_5^2)\Big]\ ,\\ e^{2\phi}&=g^2H^2\ ,\qquad \chi=\frac{i}{g}H^{-1}\ ,\\ F_{0123r}&=\frac{i}{g}\partial f^{-1}\ , \end{aligned}\tag{4.22}$$

$$f = 1 + \frac{\alpha'^2 R^4}{r^4} \ , \qquad H = 1 + \frac{r_0^4}{r^4} \ .$$

A $B$ field can be introduced as in the previous subsection. We perform T-duality in the direction $x_2$, make a rotation in the plane 2-3, and then T-duality in $x_3$. Using the standard T-duality rules, we find the following solution

$$\begin{aligned}
ds^2 &= H^{1/2}\Big[f^{-1/2}[dx_0^2 + dx_1^2 + h(dx_2^2 + dx_3^2)] + f^{1/2}(dr^2 + r^2 d\Omega_5^2)\Big] \ , \\
e^{2\phi} &= g^2 \ h \ H^2 \ , \qquad \chi = \frac{i}{g}\cos\theta \ H^{-1} \ , \\
B_{23} &= \tan\theta \ \frac{h}{f} \ H \ , \qquad A_{23} = i \sin\theta \ H^{-1} \ , \\
F_{0123r} &= \frac{i}{g}\cos\theta \ h\partial f^{-1} \ , \qquad h = \frac{f}{H\sin^2\theta + f\cos^2\theta} \ .
\end{aligned} \tag{4.23}$$

The decoupling limit of this solution is obtained by dropping the "1" in the function $f$, but not in $H$ (dropping the "1" in both functions leads to constant $h$ and $B$, that is, a solution which is essentially equivalent to the $B = 0$ case).

It is interesting to note that there is a similar solution with real gauge fields, but for a spacetime signature $(- - ++)$, which exists only in the non-commutative theory (i.e. only for $B \neq 0$). This can be obtained as follows. First, we make the shift $\theta = \alpha + \pi/2$ so that $\sin\theta \to \cos\alpha$, $\cos\theta \to -\sin\alpha$. Then we change

$$\alpha \to i\alpha \ , \qquad x_0 \to ix_0 \ , \qquad x_2 \to ix_2 \ .$$

The resulting solution is

$$\begin{aligned}
ds^2 &= H^{1/2}\Big[f^{-1/2}[- dx_0^2 + dx_1^2 + h(-dx_2^2 + dx_3^2)] + f^{1/2}(dr^2 + r^2 d\Omega_5^2)\Big] \ , \\
e^{2\phi} &= g^2 \ h \ H^2 \ , \qquad \chi = \frac{1}{g}\sinh\alpha \ H^{-1} \ , \\
B_{23} &= \coth\alpha \ \frac{h}{f} \ H \ , \qquad A_{23} = \cosh\alpha \ H^{-1} \ , \\
F_{0123r} &= \frac{1}{g}\sinh\alpha \ h\partial f^{-1} \ , \qquad h = \frac{f}{H\cosh^2\alpha - f\sinh^2\alpha} \ .
\end{aligned} \tag{4.24}$$

In this solution all fields are real. In the commutative case $B_{23} = 0$, corresponding to $\theta = 0$ in (4.23), one has $\chi = \frac{i}{g}H$, and it is not possible to make it real. This can also be seen from the fact that $B_{23} = \coth\alpha\frac{hH}{f} \neq 0$ for all $\alpha$, so the above solution (4.24) exists only for $B_{23} \neq 0$.

# 5 Potential between monopole and antimonopole

Consider a D3 brane in presence of a magnetic field $B_{23}$. The ends of a D-string attached to the D3 brane represent a monopole $m$ and an antimonopole $\bar{m}$ from the standpoint of the 3+1 dimensional non-commutative field theory. The potential between $m$ and $\bar{m}$ can be obtained in the large $N$ limit by using the dual supergravity description and computing a Wilson loop. The supergravity background we use is given by eq. (2.7) in [23]. We consider a D-string configuration in this geometry of the form $x^0 = \tau, x^2 = \sigma$, i.e. a string lying on a plane orthogonal to the magnetic field. The Born-Infeld action for the D-string in this background is

$$S = \frac{T}{2\pi\hat{g}} \int d\sigma \sqrt{(1 + a^4 u^4)(\partial_\sigma u)^2 + \frac{u^4}{R^4}} \ , \tag{5.1}$$

This action formally coincides with the case $\alpha = 0$ of eq. (5.4) in [13], representing the Nambu-Goto action for a fundamental string attached to a D3 brane in the presence of a $B_{01}$ field (this was used to obtain a quark-antiquark potential in NCOS theory). The present $m - \bar{m}$ case can be regarded as the S-dual situation. The solution to the equations of motion is

$$(\partial_\sigma u) = \frac{u^2}{R^2 \sqrt{1 + a^4 u^4}} \sqrt{\frac{u^4}{u_0^4} - 1} \ , \tag{5.2}$$

where $u_0$ represents the minimum value of $u$ reached by the string, which has its ends at $u = \infty$. It is related to the distance $L$ between $m$ and $\bar{m}$ by the formula:

$$L(u_0) = \int dx_2 = \frac{2R^2}{u_0} \int_1^\infty \frac{dy}{y^2} \frac{\sqrt{1 + a^4 u_0^4 y^4}}{\sqrt{y^4 - 1}} \ . \tag{5.3}$$

This equation defines $u_0 = u_0(L)$. The monopole-antimonopole potential $V(L)$ is obtained from

$$V(L) = V[u_0(L)] = \frac{1}{T}(S(u_0) - S(0)) \ ,$$

i.e.

$$V(L) = \frac{u_0}{\pi\hat{g}} \int_1^\infty dy \sqrt{1 + u_0^4 a^4 y^4} \left( \frac{y^2}{\sqrt{y^4 - 1}} - 1 \right) - \frac{u_0}{\pi\hat{g}} \int_0^1 dy \sqrt{1 + a^4 u_0^4 y^4} \ .$$

Being formally the same as the quark-antiquark potential of the S-dual electric non commutative theory, it will be given by the same formulas as in [13], with the appropriate correction in the $R$ (coupling) dependence:

$$u_0 = \frac{a_0 R^2}{L} + a_1 a_0^3 \frac{R^8}{L^4} + O(1/L^5) \ , \qquad R^4 = 4\pi\hat{g}N = g_{YM}^2 N \equiv \lambda \ , \tag{5.4}$$

$$a_0 = 2\int_1^\infty \frac{dy}{y^2}\frac{1}{\sqrt{y^4-1}} = \frac{2\sqrt{2}\pi^{3/2}}{\Gamma(\frac{1}{4})^2} \ , \qquad a_1 = \frac{\Gamma(\frac{1}{4})^2}{3\sqrt{\pi}} \ ,$$

$$V(L) = -\frac{4\pi N}{\lambda}\left(\frac{c_0\sqrt{\lambda}}{L} - \frac{c_1\lambda^2}{L^4}\right) + O(1/L^5) \ , \tag{5.5}$$

$$c_0 = \frac{4\pi^2}{\Gamma(\frac{1}{4})^4} \ , \qquad c_1 = \frac{16\pi^{9/2}}{3\Gamma(\frac{1}{4})^6} \ .$$

The leading term in the expansion in powers of $1/L$ coincides with the commutative case [42]. The subleading corrections are due to non-commutativity. The potential $V(L)$ can be computed numerically for all $L$. From the plot obtained in [13] for the dual quark-antiquark configuration, one can see that the curve $V(L)$ terminates at a certain $L = L_{\rm min}$. This may indicate that for $L < L_{\rm min}$ the potential becomes constant, since the only solution is that of two separate D-strings.

# 6 Conclusions

In this paper we have studied some non-perturbative aspects of NCOS theories and their OM theory origin. In particular, we have obtained monopole and dyonic solutions of (3+1) dimensional NCOS from the intersecting brane picture and found their mass density. Their stability is supported by the unbroken supersymmetries determined in sect. 4. In addition we have also presented a supergravity solution related to these non-perturbative states, and a solution representing instantons in NCSYM theory.

Compactifying OM theory on a general torus, we have shown that the full four parameter class of (3+1) dimensional NCOS theories can be obtained from toroidal compactification of OM theory, providing a geometrical interpretation for the $SL(2,Z)$ S-duality transformations of NCOS theories. We presented some evidence that there is a T-duality $SO(2,2;Z)$ group acting on NCOS theories compactified on a general non-commutative two torus, although being an *open string* theory without any closed string sector. Altogether, the full set of transformations acting on NCOS theories on $T^2$ are induced by the $SL(3,Z)\times SL(2,Z)$ U-duality group of type IIB superstring theory. A specific $SL(3)$ transformation maps NCOS theory with rational $2\pi\alpha'/V$ and $\chi = 0 = \theta^{23}$ into an ordinary Yang-Mills theory with a magnetic flux.

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